Theoretical analysis of extensive air showers V. General analysis of muons above 1000 GeV



1971 J. Phys. A: Gen. Phys. 4 276

(http://iopscience.iop.org/0022-3689/4/2/012)

J. Phys. A: Gen. Phys., 1971, Vol. 4. Printed in Great Britain

# Theoretical analysis of extensive air showers V. General analysis of muons above 1000 GeV

C. ADCOCK, R. B. COATS, A. W. WOLFENDALE and J. WDOWCZYK†

Physics Department, University of Durham, Durham, England

† Laboratory of High Energy Physics, Lodz, Poland

*MS. received* 6*th November* 1970

**Abstract.** Previous calculations by the present group—Adcock *et al.* in 1969—have been extended to yield predictions of the frequencies of multiple muons in the comprehensive experiment of Cannon and Stenerson in 1970. It is shown how comparison with experiment can give information about the magnitude of the mean transverse momentum $\langle p_t \rangle$ of the parent pions, the form of the multiplicity law and the magnitude of the primary cosmic ray intensity, all in the region of $2 \times 10^5$ GeV primary energy.

Comparison is also made with the previous experiments and the areas of agreement and disagreement are explored.

Combining all the experimental data, the average transverse momentum is somewhat less than that given by Adcock *et al.* in 1970 from an analysis of the Utah data alone and is $0{\cdot}5 \pm 0{\cdot}06$ GeV/$c$ (for the $p_t$ distribution given by Elbert *et al.* in 1968) at primary energies in the region of $2 \times 10^5$ GeV.

## 1. Introduction

Recent studies of high energy muons in extensive air showers, notably in the Kolar Gold Fields (Chatterjee *et al.* 1966, 1968, Khrishnaswamy *et al.* 1969) and in Utah (Porter and Stenerson 1969, Coats *et al.* 1970, Cannon and Stenerson 1971) have stimulated theoretical analyses of the problem with the object of deriving information of nuclear physical and astrophysical interest. In previous papers (Adcock *et al.* 1969 to be referred to as IV and Adcock *et al.* 1970 to be referred to as IVA) the present authors examined the preliminary data which had come from the Utah experiment. In the present work more extended data are analysed together with work from other experiments and attention is drawn to the manner in which the future experimental data could usefully be manipulated.

As with previous calculations a 'conservative' model for high energy collisions is adopted and calculations are made of quantities which can be directly compared with experiment. The model is basically that adopted in previous papers of the authors (e.g. de Beer *et al.* 1966 to be referred to as I) and we regard as variables the following:

(i) the mean transverse momentum $\langle p_t \rangle$, the form of the distribution in $p_t$ being assumed to be

$$N(p_t) = \frac{p_t}{p_0^2} \exp\left(-\frac{p_t}{p_0}\right)$$

(ii) the exponent $\alpha$ in the multiplicity variation, $N_s = AE_p^{\alpha}$ above $E_p = 3000$ GeV; below a primary nucleon energy of 3000 GeV the relation $N_s = 2{\cdot}7\, E_p^{1/4}$ appears to fit the available experimental data rather well and this expression is used. (In fact, in our preferred model, $\alpha = \frac{1}{4}$ throughout.)

(iii) the magnitude of the primary nucleon spectrum $j(E_p)$, although, when it is necessary to use the form of the spectrum for specific predictions, the relation adopted in IV is used (this is $j(E) = 0{\cdot}9\, E^{-2{\cdot}6}\, \mathrm{cm}^{-2}\, \mathrm{s}^{-1}\, \mathrm{sr}^{-1}\, \mathrm{GeV}^{-1}$ for $E < 2\times10^{6}$ GeV and $j(E) = 1{\cdot}3\times10^{3}\, E^{-3{\cdot}1}\, \mathrm{cm}^{-2}\, \mathrm{s}^{-1}\, \mathrm{sr}^{-1}\, \mathrm{GeV}^{-1}$ for $E > 2\times10^{6}$ GeV).

The energy spectrum for secondary pions is taken as $(N(E_\pi) = (A/T)\exp(-E_\pi/T))$ (the CKP relation—Cocconi *et al.* 1961) where $A$ is the forward multiplicity and $T$ the mean pion energy. No doubt this expression does not accurately represent the true state of affairs at very high energies and in particular it can be argued that this 'pionisation' formula does not allow for isobar production. However, a number of the predictions of relevance to extensive air shower (EAS) phenomena do not depend sensitively on the form of $N(E_\pi)$ and in any case the model does not claim to be unique: it simply represents a datum whose predictions may be compared with the experimental results. As experimental precision improves and further facets of EAS are analysed the form of $N(E_\pi)$ too will be examined.

No allowance has been made for the production of muons via the 'X-process' proposed by Bergeson *et al.* (1968, 1969).

Results of general application are given, followed by calculations which relate specifically to the conditions of the Utah experiment. Attention is directed also to a method of deriving information about the more important parameters $j(E_\mathrm{p})$, $\alpha$ and $\langle p_\mathrm{t}\rangle$ individually instead of in combination as is usually the case. Finally, direct comparison with the results of the Utah experiment presented in the previous paper (Cannon and Stenerson 1971) is made together with a brief analysis of the experimental results given by Chatterjee *et al.* (1966, 1968), Khrishnaswamy *et al.* (1969) and Barrett *et al.* (1952).

## 2. Method of calculation

The method of calculation is similar to that adopted in our previous work. For a fixed primary energy, the numbers of muons resulting from the various pion generations are summed and the moments of the radial distances are determined, from which the form of the lateral distribution is obtained. Some of these lateral distributions have been given already (in IV and IVA) for the angular range under consideration and will not be repeated. For a discussion of the calculation of the densities at large radial distances reference should be made particularly to IVA.

Geomagnetic deflexions and Coulomb scattering displacements are expected to be negligible in most situations and are not considered in the calculations.

For the calculation of expected frequencies of multiple events the probability of a shower of energy $E_\mathrm{p}$ falling at distance $r$ away from the centre of the detector area and producing the required number of muons is computed and this is then summed over all $r$ and over the primary spectrum $j(E_\mathrm{p})$.

## 3. Results of general application

### 3.1. *The mean radial distance*

An important quantity which has not been discussed in detail hitherto is the mean radial distance of the energetic muons from the shower axis. This quantity is determined largely by the mean transverse momentum and (see paper IVA) it has significance for the rate of detected multiple events in most practical cases where the size of the detection area is not large compared with the 'area' of the shower ($\simeq \pi\langle r_\mu\rangle^2$).

The variation of $\langle r_\mu \rangle$ with primary nucleon energy is given in figure 1 for several muon threshold energies and for $\langle p_t \rangle = 0{\cdot}4$ GeV/$c$. The scaling law for changes in $\langle p_t \rangle$ is quite straightforward: $\langle r_\mu \rangle \propto \langle p_t \rangle$.

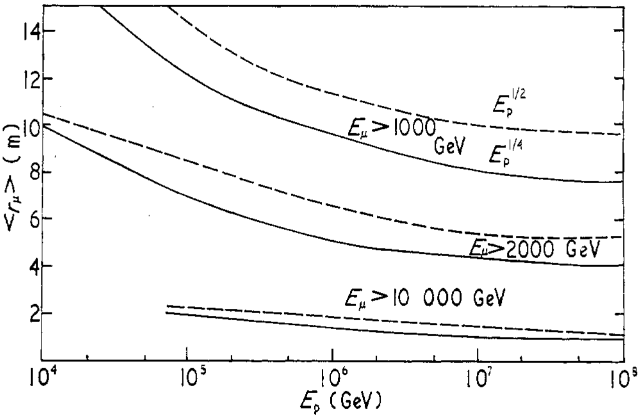


Figure 1. Mean radial distances of EAS muons from shower core. Primary protons, $\langle p_t \rangle = 0{\cdot}4$ GeV/$c$.

An explanation of the reduction of $\langle r_\mu \rangle$ with increasing $E_p$ is immediately apparent in terms of a greater yield of muons from lower altitudes occurring as significant particle generation occurs lower in the atmosphere with higher primary energies.

Figure 1 gives the results for $\theta = 60°$. Altering the zenith angle causes a simple geometrical change (variation of inclined distance to production level) and a small further change in the vertical depth from which the muons come with the result that $\langle r_\mu \rangle \propto \sec^{1{\cdot}3} \theta$ for $45° < \theta < 60°$ and $\langle r_\mu \rangle \propto E_\mu^{-0{\cdot}8}$ for $1000 < E_\mu < 2000$ GeV.

### 3.2. *Effect of detector area*

The problem of computing the expected frequency of multiple events in the case of an area $S$ over which the muon density is not constant (a common experimental

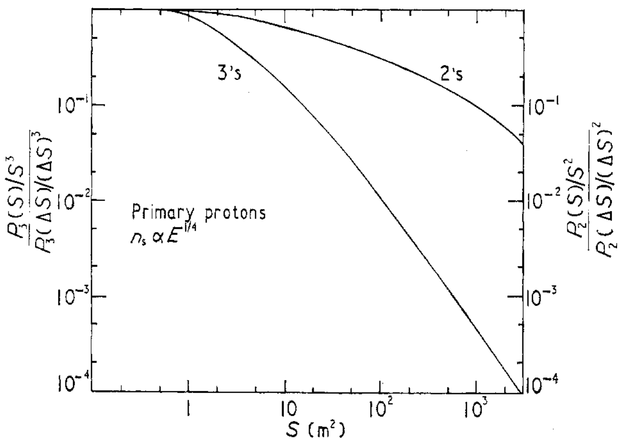


Figure 2. Area factors for twos and threes. Primary protons, $E_p^{1/4}$, $\theta = 60°$, $E_\mu$ >1000 GeV, $\langle p_t \rangle = 0{\cdot}4$ GeV/$c$. $\Delta S$ is the area of a very small detector.

condition) is not trivial. For the case of twos the frequency is proportional to $S^2$ for $S \ll \langle r_\mu \rangle^2$ and proportional to $S$ for such large areas that the whole of the shower is contained inside the area. Similarly, for threes there is a transition from $S^3$ to $S$ as $S$ increases. Calculations have been made for a particular set of parameters with the results shown in figure 2. In fact, scaling to other values of the parameters is quite straightforward using the data given in figure 1 and § 3.1, the abscissa in figure 2 being replaced by $S/k^2$ where $k$ is the ratio of the mean radius for the $E_\mu$ and $\theta$ under examination to the mean radius for the datum (1000 GeV, 60°).

## 4. Results applicable to the Utah experiment

### 4.1. *The X-process*

The problem introduced by the X-process has been mentioned briefly in § 1. The effects leading to the proposed X-process were seen in the single muon results but it is likely that the X-process will not affect the multiple muon data to any large extent for two reasons:

(i) the X-process muons will presumably be few in number for a particular shower.

(ii) their lateral distribution will very likely be wider than that of the 'pionisation' muons.

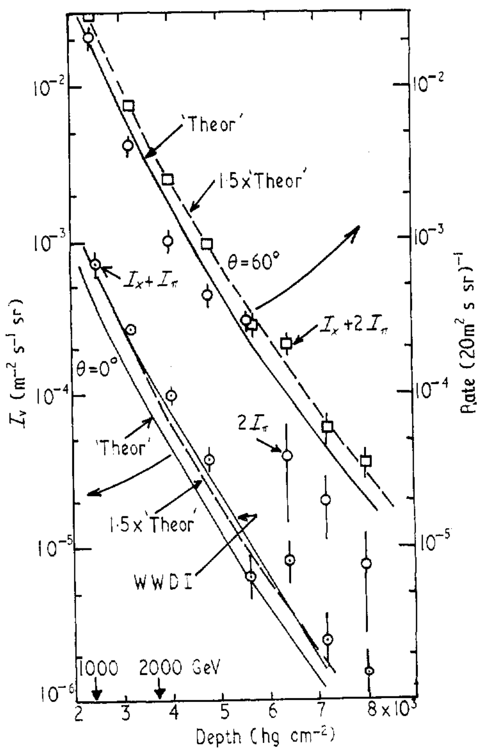


Figure 3. Vertical depth intensity curve (left-hand scale) and 60° depth intensity curve (right-hand scale). WWDI = world wide depth intensity curve (Bergeson *et al.* 1969). 'Theor' = present calculations for primary protons, $E_p^{1/4}$. $I_\pi$, $I_x$ = contribution to muon intensity from pions and from X-process according to Bergeson *et al.* (1969).

Because of the relevance of the X-process and the need, in any case, to be able to predict the 'measured' single muon spectrum, attention is first directed to this quantity. The calculated single muon spectrum at $\theta = 60°$ for $\alpha = \frac{1}{4}$ has been converted to a depth–intensity curve and is shown in figure 3 where it is compared with the Utah intensities as derived by Bergeson *et al.* (1969) (denoted by $I_x + 2I_\pi$). Also shown is that part of the intensity attributed by Bergeson *et al.* to pions alone (denoted by $2I_\pi$). It is seen that the preferred case of $\alpha = \frac{1}{4}$ gives a depth–intensity curve intermediate between the two sets of Utah data.

In a similar way the vertical depth–intensity curve for single muons has been calculated and compared with that directly measured with the result also shown in figure 3. The 'measured' line given there is the 'World Wide Depth Intensity' (WWDI) curve drawn by the Utah group through all the available data on vertical intensities from other laboratories. It is seen that the predicted curve is somewhat lower than the WWDI curve; the result of scaling up the predicted curve by a factor 1·5 is also shown—the reason for this factor is given later.

### 4.2. *Calculations of predicted rates for an area of* 20 $m^2$

4.2.1. *Rates as a function of* $E_\mu$, $\theta$ *and* $\langle p_t \rangle$. The variation of rate of multiple events with area indicated in figure 2 shows the importance of adopting a standard area to which the rates may be referred. For the Utah experiment an area of 20 $m^2$ has been adopted, this being a typical area of collection. Calculations have been made of the expected frequencies of various multiplicities as a function of angle, muon threshold energy and mean transverse momentum $\langle p_t \rangle$. Results are shown in figure 4 for the case of $\langle p_t \rangle = 0·4$ GeV/$c$ and $\theta = 60°$, this again being a typical angle in practice.

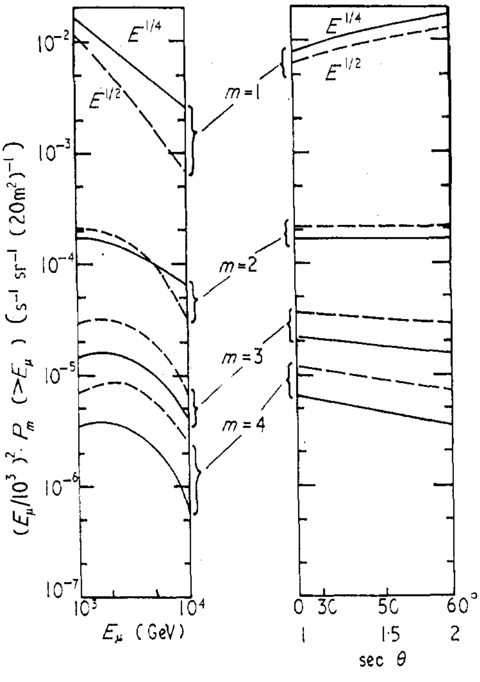


Figure 4. Expected frequencies of multiples $P_m$ against muon threshold energy $E_\mu$ (for $\theta = 60°$), and zenith angle (for $E_\mu$ >1000 GeV). Primary protons, $\langle p_t \rangle = 0·4$ GeV/$c$. Full line $E_p^{1/4}$, broken line $E_p^{1/2}$.

The ratio can be computed for different values of $\langle p_t \rangle$ using the data of figure 4 in conjunction with figure 2; rates for $\langle p_t \rangle = 0{\cdot}67$ GeV/$c$, a value which comes from IVA, are given later in figure 8.

In a similar way, the variation of rate with angle can be calculated; that for $E_\mu > 1000$ GeV and $\langle p_t \rangle = 0{\cdot}4$ GeV/$c$ is given in figure 4.

4.2.2. *Median primary energy.* The median primary energy of the detected showers has been calculated with the result shown in figure 5. It is interesting to note that in an area of 20 m$^2$ at 60° multiplicities above 3 are required before the relevant primary particles have energies above the knee in the primary energy spectrum.

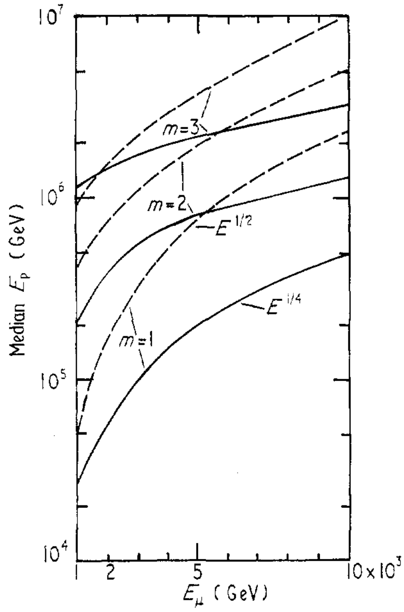


Figure 5. Median energy as a function of muon threshold energy for singles, doubles and triples. Primary protons, $\langle p_t \rangle = 0{\cdot}4$ GeV/$c$, $\theta = 60°$, 20 m$^2$ area. Full line $E_p^{1/4}$, broken line $E_p^{1/2}$.

4.2.3. *A method of deriving* $j(E_p)$, $\alpha$ and $\langle p_t \rangle$. If the three parameters referred to above are the only unknowns then there is in principle sufficient information in the Utah experiment to enable their separate values to be determined. It was demonstrated in IVA that the decoherence curve is sensitive mainly to $\langle p_t \rangle$ ($\alpha$ providing a small perturbation) and a value for this quantity was derived: $\langle p_t \rangle = 0{\cdot}67$ GeV/$c$ for $N(p_t) = (p_t/p_0{}^2)\exp(-p_t/p_0)$.

The separate values for $j(E_p)$ and $\alpha$ can be derived with reference to figures 4 and 5 by constructing curves of the type shown in figure 6.

These give the ratio of the predicted frequency of doubles (with energy above a given threshold) to that of singles with energy above that threshold energy which corresponds to the same median primary energy, as a function of the single muon value of $E_\mu$. Along each of these (full line) curves $\alpha$ varies and any point on a line corresponds to a fixed $\alpha$ and to a fixed primary energy. For example, the curve for $n = 2$ at $E_\mu = 8500$ GeV corresponds to $\alpha = \frac{1}{4}$ and the ratio of $P_2(E_\mu > 2000$ GeV) to $P_1(E_\mu > 8500\text{ GeV})$ is 1·0. Figure 5 shows that the median primary energy for both these multiplicities (singles above 8500 GeV, doubles above 2000 GeV) is approximately $4 \times 10^5$ GeV.

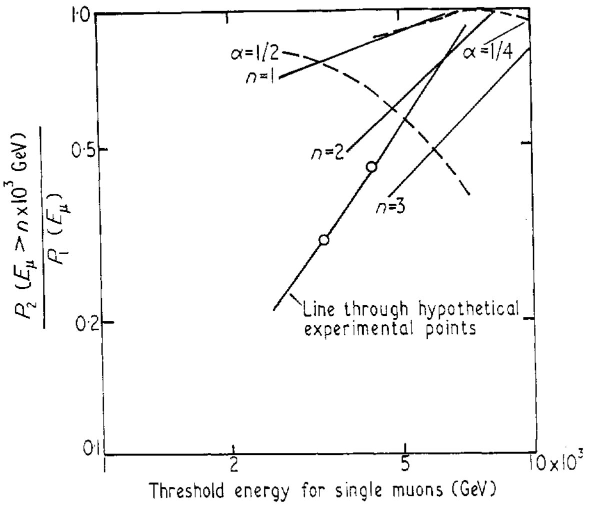


Figure 6. Ratio of frequency of doubles to singles as a function of muon threshold energy for singles. Primary protons, $\langle p_t \rangle = 0{\cdot}4$ GeV/$c$, $\theta = 60°$ (see § 4.2.3 for explanation).

The manner in which such a plot can be used is illustrated by the straight line drawn through the hypothetical experimental points. This refers to $n = 2$, that is, doubles with energy above 2000 GeV. The circles refer to singles at threshold energies below that responsible for the doubles but extrapolation to meet the $n = 2$ line gives the condition that the primary energies are the same. The value of $\alpha$ at this intersection point is then the experimentally derived value, independent of the primary intensity.

Similar curves can be plotted which relate to the ratio $P_3$ to $P_2$, $P_4$ to $P_3$ etc., and $\alpha$ values can be determined pertaining to higher primary energies (if $\alpha$ does in fact vary with $E_p$) by plotting the experimental data in a similar manner.

Having determined $\alpha$ and $\langle p_t \rangle$ (from IVA) the experimental results can be taken together with theoretical predictions to determine the primary intensity for various primary energies. The determination is by way of finding the relationship of the predicted number of events of a given multiplicity to that given in figure 4 for the appropriate $\alpha$ and by scaling the primary intensities given by the relations in § 1 accordingly. Performed in this way the calculations give the primary spectrum that would pertain if all the primaries were single nucleons.

Unfortunately, as yet, data from the Utah experiment do not cover a wide enough range of muon energies with sufficient statistical precision to enable the above method to be used. However, adequate precision is likely to be available eventually.

## 4.3. *Comparison with other theoretical predictions*

At this stage some comparison of the predictions with those of other theoretical studies can be made. A number of workers have made calculations in the high energy muon field, notably Cowsik (1966), Lal (1967), Bradt and Rappaport (1967) and

Murthy *et al.* (1968 a,b,c). A variety of high energy interaction models have been used but only Lal (1967) has calculated for the preferred model in the present work and here only for one-dimensional propagation. Over the common energy region there is agreement of the predictions to within about 20%.

## 5. Comparison with the results of Cannon and Stenerson (1971)

### 5.1. *General*

Insofar as results from a variety of depths, angles and for a variety of detecting areas were recorded in the experiment, a scaling procedure was necessary for the data grouping. As this point has been discussed in some detail in the previous paper, it is merely necessary to note here that the most important scaling, that for differing detection areas, made use of plots of the type given in figure 2.

### 5.2. *Angular variation*

Although the bulk of the angular variation in the predicted frequencies of various multiplicities comes from geometrical factors it is of value to compare observation and prediction as a check on the validity of some aspects of the model.

Representing the variation of rate with angle by $R(\theta) \propto \sec^N\theta$, $N$ has been calculated by a least-squares technique for the data given in the preceding paper for various depths. The comparison is made in figure 7, where it is seen that the agreement is adequate.

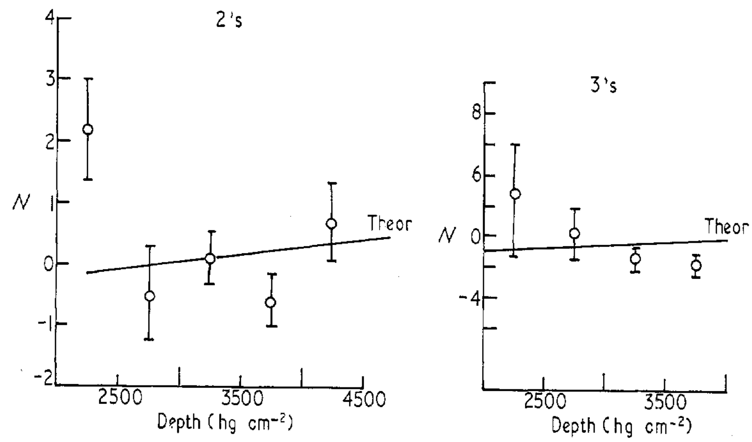


Figure 7. Angular variation of detected twos and threes. Comparison of the results of Cannon and Stenerson (1971) with the present predictions. $N$ is the exponent in the relation Rate $\propto$ $(\sec\theta)^N$.

### 5.3. *Depth variation*

The depth variation for twos and threes in 20 m at 60° is given in figure 8. Also shown are the expected curves for the case $\alpha = \frac{1}{4}$, $\langle p_t \rangle = 0{\cdot}67$ GeV/$c$, the value derived from the decoherence data (paper V). It is immediatly apparent that the predicted rates are too low and in order to obtain agreement an increase by a factor of 1·5 is required. (This normalizing factor was also required in V and there is thus consistency in the treatments.)

The effect of a similar increase on the rate of singles is shown in figure 3: there is much better agreement with the WWDI for $\theta = 0°$ and at $\theta = 60°$ the predicted intensity is close to the measured total intensity (although the meaning of this is not clear if X-production is invoked).

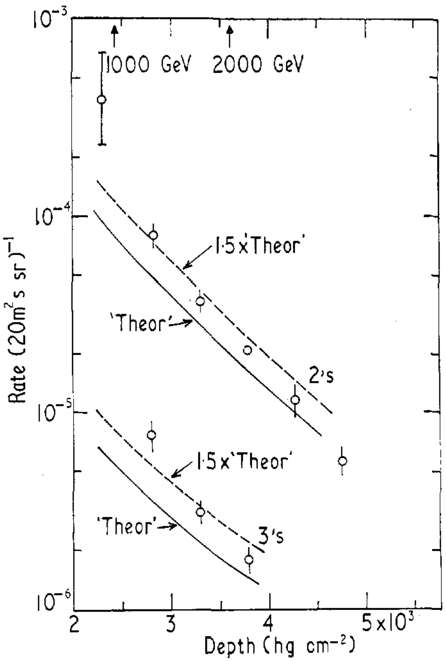


Figure 8. Depth dependence of detected twos and threes. Comparison of the results of Cannon and Stenerson (1971) with the present predictions. $A = 20\ \text{m}^2$, $\theta = 60°$, $\langle p_t \rangle = 0{\cdot}67$ GeV/$c$, $\alpha = \frac{1}{4}$.

The increase, applied universally in this way, signifies an increase in the primary spectrum $j(E_p)$ above that quoted in §1, at the energies in question ($3 \times 10^4$–$2 \times 10^6$ GeV); such an increase is not impossible.

Returning to figure 8, although there is now fairly good agreement in the region where the statistical accuracy is most precise ($E_\mu \sim 2000$ GeV) the depth dependence is not in good accord. It is likely that some, at least, of this is due to neglect of fluctuations in energy loss for the muons. A treatment incorporating such fluctuations will be attempted later.

### 5.4. *Determination of* $\alpha$

It is apparent from figure 4 that the ratio of the rates of threes to twos in 20 m$^2$ at 60° is sensitive to the value of $\alpha$. Figure 9 gives a plot of the ratio as a function of depth for the two values of $\alpha$—$\frac{1}{4}$ and $\frac{1}{2}$—and it can be seen that the ratio is almost independent of the value of $\langle p_t \rangle$. This arises because as $\langle p_t \rangle$ is increased the reduced effect of detector size relative to shower size compensates for the larger shower width. The conclusion to be drawn is that there is no evidence against the original preferred value of $\alpha = \frac{1}{4}$. Although the absolute primary intensity is not important the conclusion rests on the proviso that the slope of the primary spectrum is equal to that assumed; as can be inferred from the remarks in § 4.2.3, only when the experimental data extend over a wider range of energies, so that it is possible to compare twos and threes from the same primary energy, will it be possible to obtain $\alpha$ without this proviso.

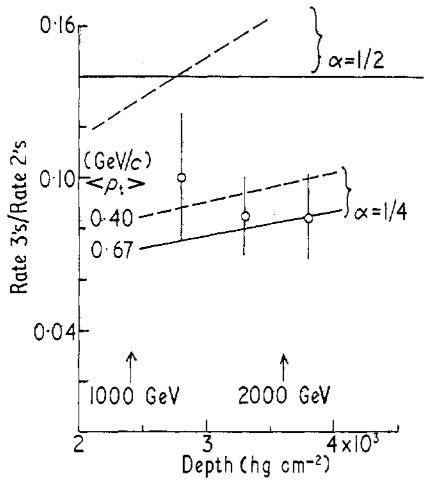


Figure 9. Determination of $\alpha$. Ratio of rates of threes to twos as a function of depth (Cannon and Stenerson 1971) compared with present predictions for $\alpha = \frac{1}{4}$ and $\alpha = \frac{1}{2}$, and two values of $\langle p_t \rangle$. $\theta = 60°$, $A = 20\ \mathrm{m}^2$.

## 6. Comparison with the results of the Kolar Gold Field experiments

### 6.1. *General*

Two experiments have been carried out in the Kolar Gold Fields, notably the study by Chatterjee *et al.* (1966, 1968) of underground muons associated with extensive air showers selected by ground level detectors, and the observations of underground muons alone by Khrishnaswamy *et al.* (1969). The latter experiment is important insofar as the observations on the angular distribution of single muons, which have been made at a similar depth to that of the experiment of Bergeson *et al.*, do not show the anomaly reported there. A comparison of the two sets of results on multiple muons is thus of importance in view of this disparity as well as the comparison of the results with the present theoretical analysis from the standpoint of deriving information about such quantities as $\langle p_t \rangle$, $\alpha$ and $j(E_p)$.

### 6.2. *The results of Chatterjee et al.* (1966, 1968)

This work comprised a measurement of the absolute number of muons of energy above 220 GeV and 640 GeV in showers of size $10^5$ to $10^6$ (primary energy $10^{15}$ to $10^{16}$ eV) and the authors have themselves derived from the results a value for $\langle p_t \rangle$. Although the method of derivation is less sophisticated than that adopted in the preceding sections of the present work the values will be adopted here in view of the difficulty in applying more rigorous methods to the conditions of particle selection (ground level shower trigger) and the fact that so far accurate calculations have not been made for such low threshold energies.

The latest value quoted for $\langle p_t \rangle$ is 0·6 to 0·7 GeV/$c$ (Chatterjee *et al.* 1968—earlier preliminary analyses gave somewhat lower values: Chatterjee *et al.* 1966).

This result comes from a measurment of the mean radius, $(20 \pm 3)$ m, of muons above a threshold of 220 GeV. It is interesting to note that extrapolating the present theoretical results to lower muon threshold energies yields a value of about 0·65 GeV/$c$ for this experiment.

It is of relevance to note that the total number of muons above 640 GeV in the size range $10^5$ to $10^6$ is close to what would be expected from the present treatment (Adcock 1970) for $\alpha = \frac{1}{4}$ and with the primary intensity equal to that adopted in § 5.3.

6.3. *The results of Khrishnaswamy et al.* (1969)

6.3.1. *Experimental details.* In view of the fact that an analysis of the multiple muons from this experiment has not been given previously, the results will be discussed in some detail.

The detector consisted of a 4 m$^2$ horizontal layer of plastic scintillator, used as the triggering element, beneath which were placed crossed layers of neon flash tubes. With this arrangement the spatial angles of tracks could be measured to within $\pm 1{\cdot}5°$. The data used in the present comparison consist of events containing two parallel muons, and were collected whilst the detector was located at a vertical depth of 1500 hg cm$^{-2}$ in the Kolar Gold Field (KGF).

6.3.2. *Analysis.* The ratio of doubles to singles in a 1 m$^2$ detector at 45° has been determined from the data in the following way. The ratios over the four 10° angular ranges from 20 to 60° have been normalized to 1 m$^2$ using figure 2 to allow for area size effects. Normalization to 45° has been made using the $(\sec\theta)^{-1{\cdot}2}$ variation for twos determined by Coats *et al.* (1970), and assuming that singles vary as $\sec\theta$ —a total factor of $(\sec\theta)^{-2{\cdot}2}$. In determining the aperture for twos it has been assumed that one muon must traverse both the scintillator and the central electrode of the bottom flash tube tray, but that the second muon need only traverse two flash tube trays (Miyake 1970 private communication). The distribution of separations of twos in the detector indicates an increase in the intensity at small separations (< 50 cm), probably due to local production, and the total number of twos has been decreased by

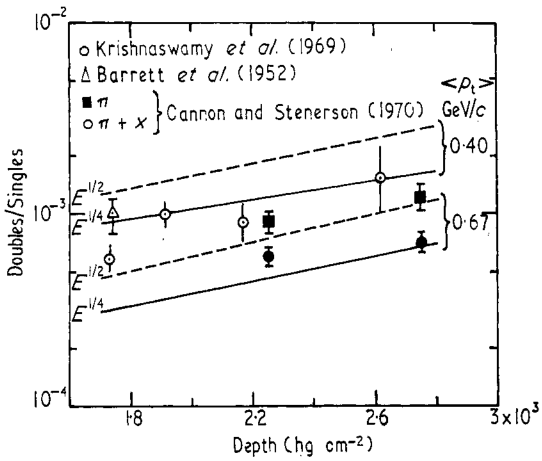


Figure 10. Ratio of rate of twos to ones at 45° in 1 m$^2$, as a function of depth for the experiments of Cannon and Stenerson (1971), Krishnaswamy *et al.* (1969), and Barrett *et al.* (1952).

10% to allow for this effect. The ratio of doubles to singles in a 1 $m^2$ detector at 45° derived in this way is shown in figure 10, as a function of depth.

6.3.3. *Comparison with prediction and with the Utah results.* Also shown in figure 10 are the predictions of the conservative model using both $\alpha = \frac{1}{4}$ and $\frac{1}{2}$ for two values of $\langle p_t \rangle$.

If it is assumed that $\alpha = \frac{1}{4}$, as follows from § 5.4, then $\langle p_t \rangle$ can be determined; the value derived from the data of Khrishnaswamy *et al.*, given in figure 10, is $\langle p_t \rangle = 0{\cdot}47 \pm 0{\cdot}1$ GeV/$c$.

Also shown in figure 10 are the Utah results on the ratio of doubles to singles, the doubles coming from the work of Cannon and Stenerson (1971) referred to earlier and the singles being derived from the data of Bergeson *et al.* (1969). The singles intensity is the total measured intensity (denoted by $\pi + x$) for one set of points, and that due to pions alone (denoted by $\pi$) for the other. For a comparison of the measured ratio of doubles to singles with that of Khrishnaswamy *et al.* the points referring to the total intensity ($\pi + x$) are relevant. It is immediately seen that the two sets are inconsistent. The discrepancy does not arise from the rate of singles—at 45° both experiments give very similar values—but rather from the rate of doubles, the Utah rates being about a factor of two lower. The reason for the difference is presumably either a technical error in one of the experiments or some physical phenomenon associated with the fact that the muon energy threshold in the Utah experiment, approximately 2 GeV, is much greater than that in the KGF experiment—approximately 0·15 GeV. The latter possibility is hard to understand without invoking yet another new phenomenon since the mean muon energy in the detected pairs should be several hundred GeV and the fraction less than 2 GeV will be quite negligible. The reason for the discrepancy is thus unresolved.

## 7. Comparison with the results of Barrett *et al.* (1952)

### 7.1. *Experimental arrangement and data*

Five counter arrangements, each of effective area approximately 0·75 $m^2$ and comprising two layers of hodoscoped geiger counters shielded by about 15 cm Pb, were operated in coincidence at a depth of 1574 hg $cm^{-2}$, and arranged such that there were ten different spacings between pairs, ranging from 1·5 to 17·5 m. After 1863 hours of running, the counters were rearranged to provide more information at separations greater than 10 m. The data are presented in figure 11 in the form of the ratio of the rate of muons in 1 $m^2$ detectors separated by $x$ to the rate of singles in one of the detectors, as a function of $x$.

### 7.2. *Analysis*

The predictions of the conservative model using $\alpha = \frac{1}{4}$ are compared with the data in figure 11 for various values of $\langle p_t \rangle$, the curves being individually normalized to give the optimum fit. At separations below 1·5 m the data are derived from a single detector, and may therefore be an overestimate of the true rate owing to the poor resolution of a single detector and also owing to local production. Although the uncertainties in the data are rather large they appear to be consistent with $\langle p_t \rangle = 0{\cdot}5 \pm 0{\cdot}1$ GeV/$c$.

The mean zenith angle is approximately 25°, corresponding to a depth of 1740 hg $cm^{-2}$. The value of $R_x/I_v$ at $x = 0$ in figure 11 is $3{\cdot}5\ 10^{-3}$, for $\langle p_t \rangle = 0{\cdot}5$ GeV/$c$. Correcting this value to 45° using the $(\sec\theta)^{-2{\cdot}2}$ variation, and allowing a factor of two—because the ways of permuting $M$ particles so that two traverse a given detector are half the permutations that put one particle in each of two detectors—the ratio of doubles to singles at 45° is shown in figure 10. This ratio is seen to be somewhat higher than measured by Khrishnaswamy *et al.* at the same depth, and the corresponding $\langle p_t \rangle$ is 0·4 GeV/$c$—a value lower than the best estimate from figure 11, although

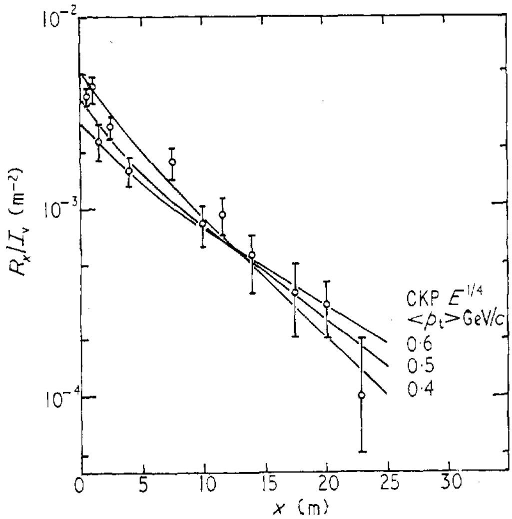


Figure 11. Data of Barrett *et al.* (1952) compared with the predictions of the conservative model. The theoretical lines have been individually normalized to the data.

the two are not necessarily inconsistent. The difference disappears if it is assumed that the intensity has been systematically overestimated; this is substantiated by the fact that the number of muons in a shower of a given primary energy is larger than theoretical prediction and other measurements—Adcock (1970).

## 8. Summary

### 8.1. *Doubles and triples in the Utah experiment*

Assuming a negligible contribution from the X-process to doubles and triples, the following points of consistency are evident from a comparison between the Utah data in 20 m$^2$ at 60° and the predictions of the conservative model ($\alpha = \frac{1}{4}$ and normalized by a factor 1·50).

(i) The rates of doubles and triples shown in figure 8 as a function of depth are in approximate agreement; some of the discrepancy is due to the neglect of fluctuations in energy loss of the muons.

(ii) In figure 7 the angular variations of the rates of doubles and triples as a function of depth are in fair agreement with theory.

(iii) The ratio of the rate of triples to the rate of doubles as a function of depth is almost independent of $\langle p_t \rangle$, and thus provides a determination of $\alpha$. Figure 9 shows that the data are quite consistent with $\alpha = \frac{1}{4}$.

### 8.2. *Singles in the Utah experiment*

The conservative model has been developed assuming only pions as the parents of the underground muons, so for a comparison with theory the experimental data must comprise only those muons derived from pions. The Utah data are considered making the following assumptions about the X-process.

8.2.1. *No X-process.* In this case all muons are assumed to come from pions. An immediate difficulty arises from the fact that the measured variation of singles with zenith angle, given by Bergeson *et al.* (1968, 1969), is less rapid than expected from the conservative theory. However, the following points of consistency can be found.

(i) In figure 3 the rate of singles (denoted by $I_x + 2I_\pi$) in 20 m$^2$ at 60° as a function of depth is not inconsistent with the theory.

(ii) In figure 10 the measured ratio of doubles to singles in 1 m$^2$ at 45° is plotted as a function of depth. The points referring to the total singles intensity (denoted by $\pi + x$ – see § 6.3.3) are the ones appropriate to the case of no X-process, and it is seen that they are not inconsistent with the theoretical predictions with $\alpha = \frac{1}{4}$ and $\langle p_t \rangle = 0{\cdot}67$ GeV/$c$.

8.2.2. *Including X-process.* Accepting now the angular variation of Bergeson *et al.*, the intensity of singles as a function of zenith angle can be fitted by the approximate expression $I = I_x + I_\pi \sec\theta$ at various depths, and the corresponding values of $I_x$ and $I_\pi$ determined. For the analysis only muons from pions must be considered and now problems arise with regard to the ratio of doubles to singles (figure 10 denoted by '$\pi$'). There it can be seen that the points for the $\pi$ component are then inconsistent with $\alpha = \frac{1}{4}$ and $\langle p_t \rangle = 0{\cdot}67$ GeV/$c$.

8.3. *Comparison with other experiments*

In the comparison between the Utah and the Krishnaswamy *et al.* data at depths greater than 2100 hg cm$^{-2}$ (figure 10, points denoted '$\pi + x$') it is found that the Utah measurement is less than the KGF data by a factor of two. At smaller depths the data of Barrett *et al.* are found to be not inconsistent with the KGF data.

## 9. Conclusions

In the Utah experiment both the decoherence measurement discussed in IVA, and the doubles and triples data are consistent with the predictions of the conservative model ($\alpha = \frac{1}{4}$, $\langle p_t \rangle = 0{\cdot}67$ GeV/$c$ for the CKP $p_t$ distribution, and a primary intensity some 50% higher than given in § 1). However, comparison with Krishnaswamy *et al.* shows the frequency of Utah doubles to be less by a factor of about two. This difference has not been resolved, but it may be related to the different zenith angles variation for singles found by the two experiments. Invoking an X-process overcomes the difficulty with the angular variation of singles in the Utah experiment, but introduces inconsistencies in a comparison between the predictions of the conservative model and the higher multiplicity data. Some of these may be eliminated if the X-process contributes significantly to the higher multiplicity data, and it is hoped to examine this in a later paper.

Confining attention to the question of the derivation of the best value of $\langle p_t \rangle$ from all the experiments, it is possible to combine the individual results giving $\langle \overline{p_t} \rangle = 0{\cdot}55 \pm 0{\cdot}07$ GeV/$c$ for the CKP distribution (and approximately $0{\cdot}5 \pm 0{\cdot}06$ GeV/$c$ for the Elbert *et al.* 1968 distribution). Thus it is likely that the best estimate of $\langle p_t \rangle$ is a little less than that quoted in IVA.

## Acknowledgments

The authors wish to thank Professors J. W. Keuffel and R. O. Stenerson for helpful correspondence and discussions. C.A. and J.W. are grateful to the Science Research Council for financial support.

## References

Adcock, C., 1970, *Ph.D. thesis*, University of Durham.

Adcock, C., Coats, R. B., Wolfendale, A. W., and Wdowczyk, J., 1970, *J. Phys.* A: *Gen. Phys.*, **3**, 697–707.

Adcock, C., Wolfendale, A. W., and Wdowczyk, J., 1969, *J. Phys.* A: *Gen. Phys.* **2**, 574–83.

Barrett, P. H., *et al.*, 1952, *Rev. mod. Phys.*, **24**, 133–78.

de Beer, J. F., Holyoak, B., Wdowczyk, J., and Wolendale, A. W., 1966, *Proc. Phys. Soc.*, **89**, 567–85.

Bergeson, H. E., *et al.*, 1968, *Phys. Rev. Lett.*, **21**, 1089–93.

—— 1969, 11*th Int. Conf. on Cosmic Rays, Budapest*, reprint MU–31.

Bradt, H., and Rappaport, S., 1967, *Phys. Rev.*, **164**, 1567–83.

Cannon, T. M., and Stenerson, R. O., 1971, *J. Phys.* A: *Gen. Phys.*, **4**, 266–75.

Chatterjee, B. K., *et al.*, 1966, *Proc. Int. Conf. on Cosmic Rays*, London (Institute of Physics), **2**, 627-31.

—— 1968, *Proc. Int. Conf. on Cosmic Rays, Calgary*, **2**, S13–16.

Coats, R. B., *et al.*, 1970, *J. Phys.* A: *Gen. Phys.*, **3**, 689–96.

Cocconi, G., Koester, L. G., and Perkins, D. H., 1961, *Lawrence Radiation Laboratory High Energy Phys. Study Seminars*, **28**, part 2, UCID–1444, 1–36.

Cowsik, R., 1966, *Proc. Int. Conf. on Cosmic Rays*, London (Institute of Physics), **2**, 656–9.

Elbert, J. W., *et al.*, 1968, *Phys. Rev. Lett.*, **20**, 124–6.

Krishnaswamy, M. R., *et al.*, 1969, 11*th Int. Conf. on Cosmic Rays*, Budapest, preprint MU–36.

Lal, S., 1967, *Nuovo Cim.*, **48**, 466–81.

Murthy, G. T., *et al.* 1968 a, *Proc. Int. Conf. on Cosmic Rays, Calgary*, **2**, S 147–152.

—— 1968 b, *Proc. Int. Conf. on Cosmic Rays, Calgary*, **2**, S 153–58.

—— 1968 c, *Proc. Int. Conf. on Cosmic Rays, Calgary*, **2**, S 159–63.

Porter, L. G., and Stenerson, R. O., 1969, *J. Phys.* A: *Gen. Phys.*, **2**, 374–91.